Navy Personnel Research and Development Center
San Diego, California 92152-7250 TN-96-5 December 1995

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A Procedure for Estimating the Probability of Detecting a Gaming Drug User

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NPRDC-TN-96-5

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Accession For	
NTIS GRA&I	☑
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

Navy Personnel Research and Development Center
San Diego, CA 92152-7250

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
December 1995

3. REPORT TYPE AND DATE COVERED
Final January 1994-July 1994

4. TITLE AND SUBTITLE
A Procedure for Estimating the Probability of Detecting a Gaming Drug User

5. FUNDING NUMBERS
Program Element: 0305889N
Work Unit: 0305889N.R2143DR001

6. AUTHOR(S)
Jules I. Borack

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Navy Personnel Research and Development Center
53335 Ryne Road
San Diego, CA 92152-7250

8. PERFORMING ORGANIZATION REPORT NUMBER
NPRDC-TN-96-5

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Chief of Naval Personnel
(PERS-63)
Navy Department
Washington, DC 20370-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

Functional Area:	Manpower
Product Line:	Information Support
Effort:	Drug Testing Algorithm

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE
A

13. ABSTRACT *(Maximum 200 words)*

In order to estimate the effectiveness of random urinalysis drug testing strategies, it is necessary to estimate the probability of detecting drug users. This report presents a series of methodologies for estimating the probability of detecting gaming drug users; individuals who choose their specific days of drug usage based upon their perceived likelihood that testing will occur on a given day. It was observed that command test strategy can dramatically impact the probability of detection. Gaming drug users were less likely to be detected under all scenarios than their non-gaming counterparts.

14. SUBJECT TERMS
Drug urinalysis, detection probability, gaming drug user

15. NUMBER OF PAGES
24

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UNLIMITED

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Foreword

This report was prepared as part of the Statistical Methods for Drug Testing project (Program Element 0305889N, Work Unit 0305889N.R2143DR001), sponsored by the Chief of Naval Personnel (PERS-63). The objective of the project is to develop a unified set of statistical methodologies for the analysis of drug testing programs and data. The work described here was performed during FY94.

This report extends the methodology developed in the reports *Markov Chains for Random Urinalysis III: Daily Model and Drug Kinetics* and *A Technique for Estimating the Conditional Probability of Detecting a Non-Gaming Drug User* to more complex patterns of drug usage.

The author thanks Dr. J. P. Boyle for his assistance in the development of this manuscript and Dr. David Blank of PERS-63 for his constructive input into the problem definition and his leadership in and dedication to Navy drug demand reduction research and development.

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Summary

Background

The U.S. Navy has maintained a zero tolerance drug policy since 1981 and has pursued an aggressive drug abuse detection and deterrence program. A major component of this effort has been a large-scale urinalysis testing program. All officer and enlisted personnel are subject to random urinalysis testing on a continuous basis. In order to estimate the effectiveness of random urinalysis drug testing strategies, it is necessary to estimate the probability of detecting drug users; that is, the probability that the user will be selected for testing and test positive.

Previous research (Thompson & Boyle, 1994; Thompson, Boyle, & Hentschel, 1993; Boyle, Hentschel, & Thompson, 1993; Evanovich 1985) has developed Markov models for analyzing random urinalysis testing strategies. In particular, Thompson and Boyle (1994) presented daily Markov models that include drug excretion rate kinetics. Borack (1995) extended these efforts to include more complex patterns of drug usage by non-gaming individuals; that is, users who choose their specific days of drug usage without regard to command urinalysis drug testing strategy.

An important segment of drug users consists of gaming individuals: persons who choose specific days of drug usage based upon their perceived likelihood that testing will occur on a given day. This effort is concerned with developing techniques for estimating the probability of detecting gaming drug users.

Objective

The objective of this effort is to develop a series of methodologies for estimating the probability of detecting gaming drug users under a wide variety of scenarios, which differ by frequency of drug use and period of time the drug remains detectable (i.e., wear-off period).

Approach

Three algorithms were developed based upon increasingly less restrictive assumptions concerning the impact of drug use and wear-off on the conditional probability of drug detection; that is, the probability that the individual will test positive if selected for drug testing. The simplest case assumes that the individual tests positive whenever a drug test is administered during the drug wear-off period. A less stringent case assumes the conditional probability of detection is based upon time since most recent drug use. The third, and most general scenario, assumes that the conditional probability of detection depends upon the pattern of use prior to testing. Specific command testing strategies are modeled to determine the probability of detecting a gaming drug user during a given period (e.g., week).

Results

Analysis of the three methodologies revealed that it is possible to combine the least restrictive approach with an application of the geometric probability distribution to build models to estimate both the probability of detection during a period (month) and the average duration (in months) until detection. For non-gaming drug users, it was observed that the probability of detection and the

expected duration until detection are approximately proportional to the command monthly test rate. Under certain conditions, this is also true for gaming users. Under these conditions, a command that tests an average of 20% of its personnel monthly can expect to detect approximately double the number of its users within a testing month (and detect them in half as many months) as a command that tests an average of 10% of its personnel monthly. When compared to non-gaming users, gaming users will generally avoid detection for longer periods of time. The magnitude of this difference depends upon both the command drug testing strategy and monthly testing rate, as well as the frequency of drug use by the individual.

Conclusions and Recommendations

The methodologies presented in this report provide a useful approach for analyzing the probability of detecting gaming drug users. The analyses show that command monthly test rates and strategies can have a dramatic impact on the expected number of months until detection of gaming drug users.

It is recommended that this approach be used as part of the Navy's Drug Policy Analysis System (DPAS). Additionally, it is recommended that models be developed for estimating the cost to the Navy of an undetected drug user as a function of the length of time undetected. These costs, in conjunction with methodologies developed in this report, will be an essential input to the formulation of models to assist in the development of an optimal Navy drug testing program.

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Introduction

The U.S. Navy has maintained a zero tolerance drug policy since 1981 and has pursued an aggressive drug abuse, detection, and deterrence program. A major component of this effort has been a large-scale urinalysis testing program. All officer and enlisted personnel are subject to random urinalysis testing on a continuing basis. Current policy (Chief of Naval Operations [CNO], 1994) requires Navy commands to test 10 to 30% of their members every month. The testing program is intended to deter and detect drug abuse (as well as provide data on the prevalence of drug abuse), and has been successful in reducing the proportion of individuals testing positive for drugs from 7% in 1983 to 1% in 1991.

Previous reports (Thompson & Boyle, 1994; Thompson, Boyle, & Hentschel, 1993; Boyle, Hentschel, & Thompson 1993, Evanovich, 1985) developed Markov models for analyzing random urinalysis strategies. In particular, Thompson and Boyle (1994) presented daily Markov models that included drug excretion rate kinetics. These models allow for a fixed length cycle (e.g., weekly, monthly) and treat initial drug dose as a random variable. The models considered drug detection time in urine and noted that detection time varies by specific drug, dose, physical condition, fluid intake, and frequency of ingestion (Ambre, Ruo, Nelson, & Belknap, 1988; Beckett & Rowland, 1965; Hamilton, Wallace, Shimek, Land, Harris, & Christenson, 1977; Johansson, Gillespie, & Halldin, 1990; Johansson & Halldin, 1989; and Cook, Jeffcoat, Sadler, Hill, Voyksner, Pugh, White, & Perez-Reyes, 1992). Borack (1994) extended these efforts to include more complex patterns of drug usage by non-gaming individuals; users who choose their specific days of drug usage without regard to command urinalysis drug testing strategy.

In order to estimate the effectiveness of random urinalysis drug testing strategies, it is necessary to estimate the probability of detecting drug users. An important segment of drug users consists of gaming individuals: users who choose specific days of drug usage based upon their perceived likelihood that testing will occur on a given day. This report extends previous research by developing methodologies for estimating the probability of detecting gaming drug users. Three algorithms were developed based upon increasingly less restrictive assumptions about the relationship between patterns of drug use and the conditional probability of drug detection; that is, the probability that the individual will test positive if selected for drug testing. The simplest case assumes that drugs remain in the system over some time frame and the individual tests positive with certainty whenever a drug test is administered during this wear-off period. A less stringent case assumes the conditional probability of testing positive is based upon time since most recent drug use. The third, and most general scenario, assumes the conditional probability of testing positive depends upon the pattern of drug use prior to testing. When these conditional probabilities are modeled in conjunction with command testing strategies, estimates of the probability of detecting a gaming drug user and the average time until detection can be obtained.

Objective

The objective of this effort is to develop a series of methodologies for estimating the probability of detecting gaming drug users under a wide variety of drug usage and wear-off scenarios.

Methodology

Three algorithms that simulate a gaming users strategy for selecting days on which to use drugs will be developed based upon alternative assumptions about drug wear-off. The first scenario assumes that the individual tests positive with certainty during the drug wear-off period; the second assumes that the likelihood of detection depends upon the time since drugs were last used; and the third assumes that the likelihood of detection depends upon the pattern of drug use during the W days preceding the test day. The algorithms all assume that the gaming user plans to use drugs on a specific number of days during a period (e.g., 2 days during a week; 4 days during a 30-day month) and knows which days will be testing days. The probability of selection is the same on all test days. The objective of the user is to minimize the probability of detection during the period. The algorithms choose drug use days sequentially by selecting days that add the least additional probability of detection.

Let $\{K\}$ represent a $1 \; x \; D$ vector of drug test days where the ith element, k_i is initially binary-valued and coded 1 or 0 depending on whether or not the day will be a drug test day;[1] and D represents the number of days in the period. Let $T = \sum_i k_i$ be the number of days during the period on which testing will occur. For example, if testing will take place on Wednesday and Saturday of a given week, then $K = [0\ 0\ 0\ 1\ 0\ 0\ 1]$ where the first element of K represents Sunday. In this case, $T = 2$. Let $\{U\}$ represent a $1 \; x \; D$ vector of drug use days where the ith element, u_i, is binary-valued and coded 1 or 0 depending on whether drugs are used on that day. Let $R = \sum_i u_i$ be the total number of days on which drugs will be used. For example, if the individual uses drugs only on Tuesday, then $U = [0\ 0\ 1\ 0\ 0\ 0\ 0]$ and $R = 1$. Let $W \leq D$ represent the wear-off period (in days) of the drug. Let $\{A_u\}$ represent a $1 \; x \; D$ vector where the ith element α_{ui} denotes the conditional probability of testing positive on day i (i.e., the probability of testing positive given the individual has been selected for testing) with drug use profile $\{U\}$. Note that $\alpha_{ui} = f\{U\}$ depends only upon the drug usage during the W days prior to day i; that is, only upon $[u_{i-W}, u_{i-W+1}, ..., u_{i-1}]$. For example, if the drug user in our example tests positive with certainty one day after using drugs (on Tuesday) and has a probability of .5 of testing positive on the second day after drug use (but will not test positive beyond the second day), then $W = 2$ and $\{A_u\} = [0\ 0\ 0\ 1\ .5\ 0\ 0]$. The objective of the user is to minimize the probability of testing positive during the period, P_D, where $P_D \propto \sum_i k_i \alpha_{ui}$, the number

[1]During the updating process, the elements of $\{K\}$ represent the additional risk of detection. For scenario 2, they represent indicators of the time since drugs were last used.

of days the user is at risk for detection. The individual is at risk for detection whenever testing occurs within W days after drug use. For our hypothetical user, $\sum_i k_i \alpha_{ui} = 1$; if our user is selected for testing on Wednesday, then detection will occur. (If testing were to also occur on Thursday, then $\sum_i k_i \alpha_{ui} = 1.5$; that is, the individual would be at risk for detection on both Wednesday and Thursday, but the probability of detection on Thursday is only .5 if the individual is selected for testing.) If this user's command tests equally on Wednesday and Saturday at a rate of 5% per week (i.e., 2.5% on each test day), then this user (who is obviously not effectively gaming the system) has a probability of 2.5% of being detected during the week.

Let $\{M\}$ represent a $D \, x \, (W + 3)$ matrix with elements defined as follows:[2]

$$m_{ij} = \begin{cases} k_{i+j} & 1 \le j \le W \\ \sum_{j=1}^{W} m_{ij} & j = W = 1 \\ \underbrace{min\,(j) \in k_{i+j} > 0}_{i<j} & j = W = 2 \\ u_i & j = W = 3 \end{cases} \tag{1}$$

Here, the *ith* row of the matrix represents the impact of using drugs on the *ith* day of the period. The first W elements of the *ith* row represent whether a day is a risk day (that is, a day when the individual could be tested for drugs and caught), the *W+1st* element represents the total number of additional days of risk resulting from using drugs on the *ith* day, the *W+2nd* element represents the number of days until the next test day occurs within the wear-off period (0 if there is no test day within the wear-off period), and the last element is an indicator variable denoting whether the day was selected as a drug use day.[3] For the example under discussion, the first row represents the impact of using drugs on Sunday, the second row represents Monday, etc. Columns one and two indicate whether the first and second days of the wear-off period, respectively, are risk days. Thus, these columns represent Monday and Tuesday for row 1 (day 1: Sunday), Tuesday and Wednesday for row 2 (day 2: Monday), etc. Column 3 represents the sum of the values in columns 1 and 2 and column 4 indicates the number of days until the next test day. For example, on Sunday (row 1), the number of days until a test day occurs is coded as 0 since testing does not occur on Monday or Tuesday. On Monday (row 2), the number of days until a test day occurs (within the wear-off period) is 2 since the next test day is Wednesday. The last column, column 5, indicates whether a day was actually chosen as a day for drug use. In this example, only Tuesday (row 3) was selected. Thus, for the example under discussion:

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[2] For $j = W + 1$, $\sum_{j=1}^{W} m_{ij} = \sum_{j=1}^{W} k_{i+j}$. For consistency of notation throughout the scenarios, $\sum_{j=1}^{W} m_{ij}$ will be used.

[3] This formulation assumes a cyclic period in order to capture the essence that the beginning of a month is influenced by drug testing policy and drug use during the previous month. Therefore, the days following the end of the period are the first days of the current period. Thus, for a 28-day period, a drug with a 5-day wear-off period taken on the 27th day would be detectable on days 28, 1, 2, 3, and 4.

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that this user is at risk for detection on one day, Wednesday, the first day after drug use.

Scenario 1: The user tests positive with probability equal to 1 whenever testing is conducted within W days after drug consumption. That is, $\alpha_{ui} = 1$ when $\sum_{q=i-W}^{i-1} u_q \geq 1$; 0 otherwise. The algorithm for selecting days for drug use so as to minimize the probability of detection during the period is as follows:

- Select first day of drug use, i_1, where (in order):

  1. $m_{i_1, W+3} = 0.$

  2. $m_{i_1, W+1} = \underbrace{min}_{i} \quad m_{i, W+1}.$

  3. $m_{i_1, W+2} = \underbrace{max}_{i} \quad m_{i, W+2}.$

Thus, the algorithm selects the (first) drug use day from those days that had not already been picked (here, all) by choosing the day that adds the least additional risk of detection. In case of ties, the algorithm chooses the day with the least additional risk that is furthest from the next test day during the wear-off period.

- Update $\{K\}$ as follows:

  1. $k_{i_1+j} = 0 \qquad 1 \leq j \leq W.$

  2. $k_i = k_i$ otherwise.

- Update $\{M\}$as in (1).

Repeat until R drug use days have been selected.

Whenever a day has been selected for drug use, the user will test positive with certainty over the following W days. As a result, no additional risk of detection is possible on these W days from additional days of drug use. This can be represented as $k_{i_1+j} = 0 \quad 1 \leq j \leq W$; that is, the user can

assume no additional testing occurs on these W days. In our example, if drugs are used on Tuesday, then Wednesday and Thursday are potential risk days and will be counted as such. No additional risk of detection on Wednesday and Thursday is possible no matter which additional days of drug use are selected. Therefore, selection of another day for drug use can assume Wednesday and Thursday represent no additional risk of detection, or equivalently, are not test days. Scenarios 1a, 1b and 1c in the results section illustrate the application of this algorithm.

Scenario 2: Drug wear-off is non-unitary; that is, $a_{ui} = p_j \quad (0 \le p_j \le 1) \quad (1 \le j \le W)$ for the j days following the last day of drug usage; $\alpha_{ui} = 0$ elsewhere. In general, p_j are monotonically non-increasing. Define $\{K\}$ as above and $\{M\}$ as follows:

$$m_{ij} = \begin{cases} p_j k_{i+j} & 1 \le j \le W \\ \sum_{j=1}^{W} m_{ij} & j = W+1 \\ \underbrace{min(j) \in k_{i+j} > 0}_{i<j} & j = W+2 \\ u_i & j = W+3 \end{cases} \tag{2}$$

Note that the daily risk has been modified by the less than unitary conditional probability of detection. The algorithm for this scenario is as follows:

- Select first day of drug use, i_1, where (in order):

1. $m_{i_1, W+3} = 0.$

2. $m_{i_1, W+1} = \underbrace{min}_{i} \; m_{i, W+1}.$

3. $m_{i_1, W+2} = \underbrace{max}_{i} \; m_{i, W+2}.$

As in scenario 1, the algorithm selects the day that adds the least additional risk of detection from among those days that had not already been selected (here, all). Ties are again broken by choosing the day furthest from the next testing day within the wear-off period.

- Update $\{K\}$ as follows:

1. $k_{i_1+j} = j+1 \quad 1 \le j \le W$ and $p_j \ge k_{i_1+j-1}$ and $k_i \ne 0.$

2. $k_i = k_i$ otherwise.

This adjusts the maximum risk of detection for those days in the wear-off period. Here, a value greater than 1 indicates that the day has already been influenced by previous drug use; in particular, drugs were used k_{i_1+j-1} days previously. For example, if Tuesday is selected as a drug use day, then

Wednesday would be coded as 2, which indicates that drugs were used 1 day previously, and Thursday would be coded as 3 indicating that drugs were used 2 days previously.

- Update $\{M\}$ as follows:

$$m_{ij} = \begin{cases} p_j k_{i+j} \text{ if } k_{i+j} = 0 \text{ or } 1, \text{ else} = \min\{0, p_j - p_{k_{i+j-1}}\} & 1 \le j \le W \\ \sum_{j=1}^{W} m_{ij} & j = W+1 \\ \underbrace{min(j) \in k_{i+j} > 0}_{i<j} & j = W+2 \\ u_i & j = W+3 \end{cases} \quad (3)$$

Repeat until R days of drug usage have been selected.

Here, the logic of the algorithm is that additional risk of detection depends upon the difference between the probabilities of detection based upon date of last use. For example, if both Wednesday and Thursday were test days, the selection of Wednesday as a drug use day (in addition to Tuesday) would increase the conditional probability of detection on Thursday by $p_1 - p_2$. In our previous example, $p_1 = 1$ and $p_2 = .5$; therefore, the selection of Wednesday as a drug use day would add an additional conditional probability of selection of .5 on Thursday. Prior to selection of Wednesday as a drug use day, the conditional probability of detection had been 1 on Wednesday and .5 on Thursday. After the selection of Wednesday as a drug use day, the conditional probability of detection on Thursday had risen to 1. Thus, drug use on Wednesday adds an additional conditional probability of selection of .5 on Thursday. Examples illustrating the use of this algorithm are presented as scenarios 2a, 2b, and 2c in the results section.

Scenario 3: Drug wear-off is non-unitary and conditional probabilities accumulate. For example, if drugs are used on two consecutive days before a testing day (assuming a two day wear off), i, then the conditional probability of detection, α_{ui}, is $p_1 + p_2(1 - p_1) = 1 - (1 - p_1)(1 - p_2)$; that is, we add to p_1 the additional probability of detection if drug use on day i-1 goes undetected but drug use on day i-2 is detected. In general $a_{ui} = 1 - \prod_k (1 - p_k)$ where k are determined from the non-zero elements of $[u_{i-w}, u_{i-w+1}, \ldots, u_{i-1}]$.

Define $\{K\}$ as above and $\{M\}$ as follows:

$$m_{ij} = \begin{cases} p_j k_{i+j} & 1 \le j \le W \\ \sum_{j=1}^{W} m_{ij} & j = W+1 \\ \underbrace{min(j) \in k_{i+j} > 0}_{i<j} & j = W+2 \\ u_i & j = W+3 \end{cases} \quad (4)$$

Note that the daily risk has again been modified by the less than unitary conditional probability of detection. The algorithm for selection of drug use days under this scenario is as follows:

- Select first day of drug use, i_1, where (in order):

1. $m_{i_1, W+3} = 0$.

2. $m_{i_1, W+1} = \underbrace{min}_{i} \quad m_{i, W+1}$.

3. $m_{i_1, W+2} = \underbrace{max}_{i} \quad m_{i, W+2}$.

This parallels the procedure in scenarios 1 and 2.

- Update $\{K\}$ as follows:

1. $k_{i_1+j} = k_{i_1+j}(1-p_j) \quad 1 \le j \le W$.

2. $k_i = k_i$ otherwise.

This adjusts the maximum risk of detection for those days in the wear-off period. Here, the remaining risk is modified to equal the probability that previous drug use went undetected.

- Update $\{M\}$ as in (4).

Here, successive iterations reduce the incremental risk by the probability of detection based on previous drug use. Note that scenario 1 is a special case of scenario 3 where $p_j = 1 \, (1 \le j \le W)$. Examples illustrating the use of this algorithm appear as examples 3a, 3b, and 3c in the results section.

Results

The three algorithms were applied to a number of drug use and testing scenarios. The probability of detecting a gaming drug user during a 7-day period was compared to the probability of detecting a non-gaming individual using the methodology described in (Borack,1995). Three command weekly testing programs were considered and are listed in Table 1. All programs test an average of two days per week; some test on days 1 and 2, some on days 1 and 4, and some decide whether a day is a test day randomly with probability $\frac{2}{7}$; that is, each day has two chances in seven of being selected. Some programs use a .025 weekly test rate (which approximates a .10 monthly test rate) while others use a .050 weekly test rate (which approximates a .20 monthly test rate). The weekly test rates of .025 and .05 imply daily test rates of .0125 and .025, respectively, since 2 days of testing occur each week. For the gaming user, however, the random selection of days requires that all days be viewed as potential test days with daily probability of selection equal to $\frac{w_r}{7}$, where w_r represents the weekly test rate.

Table 1

Testing Strategies and Wear-off Assumptions

Scenario	Wear-off	Strategy	Weekly Test Rate
1a	3-day	Test on days 1, 2	.025; .05
1b	"	Test on days 1, 4	.025; .05
1c	"	All days possible	.025; .05
2a	Time since use	Test on days 1, 2	.025; .05
2b	"	Test on days 1, 4	.025; .05
2c	'	All days possible	.025; .05
3a	Cumulative	Test on days 1, 2	.025; .05
3b	"	Test on days 1, 4	.025; .05
3c	"	All days possible	.025; .05

Three drug wear-off scenarios for cocaine were considered. The first scenario assumed certainty of detection up to three days after use; the second assumed the probability of detection depends on time since last use; and the third assumed the impact of drug usage is cumulative as described in scenario 3 of the previous section. The probabilities to be used in scenarios 2 and 3 were estimated by Thompson and Boyle (1994) and are listed in Table 2.

Table 2

Probabilities of Testing Positive for Cocaine Use 1 to 7 Days After Ingestion

Day	Probability
1	.9376
2	.6508
3	.1229
4	0
5	0
6	0
7	0

Figure 1 presents the initial values of $\{K\}$ corresponding to scenarios 1a, 2a, 3a; 1b, 2b, 3b; and 1c, 2c, 3c. Figures 2 and 3 present the initial values of $\{M\}$ corresponding to scenarios 1a, 1b, 1c, and 2a, 2b, 2c, respectively. Note that the initial values of $\{M\}$ for scenarios 3a, 3b, 3c are equivalent to the corresponding matrices for scenarios 2a, 2b, 2c.

$$\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}1\\0\\0\\1\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\end{bmatrix}$$

Scenario 1a, 2a, 3a — Scenario 1b, 2b, 3b — Scenario 1c, 2c, 3c

Figure 1. Initial values of $\{K\}$ for command drug testing scenarios.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 0 & 2 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \end{bmatrix}$$

Scenario1a Scenario1b Scenario1c

Figure 2. Initial values of $\{M\}$ for scenarios 1a, 1b, and 1c.

$$\begin{bmatrix} .94 & 0 & 0 & .94 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & .12 & .77 & 2 & 0 \\ .94 & .65 & 0 & 1.59 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & 0 & .65 & 2 & 0 \\ .94 & 0 & 0 & .94 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & 0 & .65 & 2 & 0 \\ .94 & 0 & 0 & .94 & 1 & 0 \end{bmatrix} \begin{bmatrix} .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \end{bmatrix}$$

Scenario2a/3a Scenario2b/3b Scenario2c/3c

Figure 3. Initial values of $\{M\}$ for scenarios 2a, 2b, 2c; 3a, 3b, and 3c.

The first day of drug use selected by the gaming user under each of the scenarios in Table 1 is presented in Table 3. These days were determined from the values in columns 4 and 5 of the corresponding $\{M\}$.

Table 3

User Selection of First Day of Drug Use

Scenario	Wear-off	Strategy	Weekly Test Rate
1a	3-day	Test on days 1, 2	2
1b	"	Test on days 1, 4	4
1c	"	All days possible	1
2a	Time since use	Test on days 1, 2	2
2b	"	Test on days 1, 4	4
2c	'	All days possible	1
3a	Cumulative	Test on days 1, 2	2
3b	"	Test on days 1, 4	4
3c	"	All days possible	1

The algorithm next updates the corresponding values of $\{K\}$. The updated vectors for each scenario are presented in Figure 4. Note the changes relating to scenarios 1c, 2c, and 3c. The values of $\{K\}$ for scenario 1c indicate that no additional risk of detection can occur on days 2, 3, and 4 since the user is now vulnerable to detection on these days. The updated values of $\{K\}$ for scenario 2c indicate that drugs were last used 1 day before day 2, 2 days before day 3, and 3 days before day 4. The updated values of $\{K\}$ for scenario 3c show that the additional risk of detection on day 2 has been reduced to .06, on day 3 to .35, and on day 4 to .88 since these are the probabilities of going undetected even though drugs had been used 1, 2, or 3 days ago, respectively.

$$\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\\1\\1\\1\end{bmatrix}$$

Scenario
1a, 1b, 1c

$$\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\2\\3\\4\\1\\1\\1\end{bmatrix}$$

Scenario
2a, 2b, 2c

$$\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\\0\\0\\0\end{bmatrix}\begin{bmatrix}1\\.06\\.35\\.88\\1\\1\\1\end{bmatrix}$$

Scenario
3a, 3b, 3c

Figure 4. Updated values of $\{K\}$ for command drug testing scenarios.

Figures 5, 6, and 7 present the updated values of all scenarios. The updates to the a and b scenarios required only that a day be noted as having already been selected for drug use; updates to the c scenarios required calculation of revised detection probabilities.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 0 & 2 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 3 & 1 & 1 \\ 0 & 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Scenario 1a Scenario 1b Scenario 1c

Figure 5. Updated values of {*M*} for scenarios 1a, 1b, and 1c after selection of first day of drug use.

$$\begin{bmatrix} .94 & 0 & 0 & .94 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & .12 & .77 & 2 & 0 \\ .94 & .65 & 0 & 1.59 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & 0 & .65 & 2 & 0 \\ .94 & 0 & 0 & .94 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & .12 & .12 & 3 & 0 \\ 0 & .65 & 0 & .65 & 2 & 0 \\ .94 & 0 & 0 & .94 & 1 & 0 \end{bmatrix} \begin{bmatrix} .94 & .65 & .12 & 1.71 & 1 & 1 \\ .29 & .53 & .12 & .94 & 1 & 0 \\ .82 & .65 & .12 & 1.59 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & .12 & 1.71 & 1 & 0 \\ .94 & .65 & 0 & 1.59 & 1 & 0 \\ .94 & 0 & 0 & .94 & 1 & 0 \end{bmatrix}$$

Scenario 2a Scenario 2b Scenario 2c

Figure 6. Updated values of {*M*} for scenarios 2a, 2b, and 2c after selection of first day of drug use.

$$
\begin{bmatrix}
.94 & 0 & 0 & .94 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .12 & .12 & 3 & 0 \\
0 & .65 & .12 & .77 & 2 & 0 \\
.94 & .65 & 0 & 1.59 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & .12 & .12 & 3 & 0 \\
0 & .65 & 0 & .65 & 2 & 0 \\
.94 & 0 & 0 & .94 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & .12 & .12 & 3 & 0 \\
0 & .65 & 0 & .65 & 2 & 0 \\
.94 & 0 & 0 & .94 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
.94 & .65 & .12 & 1.71 & 1 & 1 \\
.33 & .57 & .12 & 1.02 & 1 & 0 \\
.83 & .65 & .12 & 1.60 & 1 & 0 \\
.94 & .65 & .12 & 1.71 & 1 & 0 \\
.94 & .65 & .12 & 1.71 & 1 & 0 \\
.94 & .65 & .01 & 1.60 & 1 & 0 \\
.94 & .04 & .04 & 1.02 & 1 & 0
\end{bmatrix}
$$

Scenario 3a | Scenario 3b | Scenario 3c

Figure 7. Updated values of {*M*} for scenarios 3a, 3b, and 3c after selection of first day of drug use.

From the values appearing in columns 4, 5, and 6 of the matrices in Figures 4 through 7, the second day of drug use can now be obtained. The selected days are listed in Table 4. In these examples, command test strategy significantly impacted the choice of drug use days. The wear-off scenarios did not materially impact the selection of drug use days since all assumed a three-day wear-off period. The scenarios did, however, impact the probability of detection during the period. Based upon these optimal drug use strategies, Table 5 presents the probability of detection during the week for each scenario.[4] The values for scenario 3c were derived from the conditional probability of detecting users with cumulative patterns of drug use as found in Table 2 of Borack,1995. Command test strategy exerted a profound impact upon the likelihood of detecting this gaming drug user. Testing on the first two days of the week proved to be an easily evaded strategy while random selection of test days yielded the highest probability of detection. Spacing of drug test days as in the b-series of scenarios yielded results somewhere between these two extremes. This strategy would be relatively less successful if comparisons were based on drugs with smaller wear-off periods.

Table 6 provides the average duration (in weeks) until detection of this gaming user under each of the testing and wear-off scenarios. Based upon properties of the geometric distribution (Feller, 1957), the average duration can be calculated as $\frac{1-PDET}{P(DET)} + .5$ where $P(DET)$ denotes the probability of detection. The mean of the geometric distribution has been incremented by .5 based on the assumption that detection occurs, on average, at the mid-point of the week. Using the same number of expected days of testing, it is clear that command test strategy dramatically impacts the average time until the gaming drug user is detected. For example, a weekly test rate of .05 provides no assistance in detecting our gaming drug user when tests are conducted on the first two days of the week, but makes the user considerably vulnerable to detection when all days are candidates for selection as drug testing days.

[4]For a given potential test day, the probability of evading detection = 1 - probability of detection = 1 - *P* (day selected as test day) x *P* (individual is selected | day is selected) x *P* (individual tests positive | individual is selected for testing). Probability of detection during the week = 1 - probability of evasion on each test day = 1 - *P* (evasion on day 1) x (*P* evasion on day 2) .... *P* (evasion on day 7).

Table 4

User Selection of Two Days of Drug Use

Scenario	Wear-off	Strategy	Weekly Test Rate
1a	3-day	Test on days 1, 2	2, 3
1b	"	Test on days 1, 4	4, 1
1c	"	All days possible	1, 2
2a	Time since use	Test on days 1, 2	2, 3
2b	"	Test on days 1, 4	4, 1
2c	'	All days possible	1, 2
3a	Cumulative	Test on days 1, 2	2, 3
3b	"	Test on days 1, 4	4, 1
3c	"	All days possible	1, 2

Table 5

Probability of Detecting Gaming Drug User During the Week

	Weekly Test Rate	
Scenario	.025	.05
1a	0	0
1b	.0125	.0250
1c	.0142	.0283
2a	0	0
2b	.0015	.0030
2c	.0094	.0188
3a	0	0
3b	.0015	.0030
3c	.0099	.0198

Table 6

Average Duration (in weeks) Until Detection of Gaming User

	Weekly Test Rate	
Scenario	.025	.05
1a	Infinite	Infinite
1b	79.5	39.5
1c	69.9	34.8
2a	Infinite	Infinite
2b	666.2	332.8
2c	105.9	52.7
3a	Infinite	Infinite
3b	666.2	332.8
3c	100.5	50.0

To illustrate the impact of gaming, consider scenarios 1a-c. Using the methodology discussed in scenario 1 of Borack,1995, the probability of detecting a similar non-gaming user during the week $\cong .857\ w_r$ where w_r represents the weekly testing rate. For $w_r = .025$, the probability of detection during the week is .0214 and the average number of weeks until detection is 46.17; for $w_r = .05$, the corresponding values are .0428 and 22.84. Thus, the gaming user can substantially reduce the risk of detection under all scenarios, although the reduction is of smaller magnitude when all days are candidates for selection selected as test days.

Conclusions and Recommendations

The methodologies presented in this report provide a useful approach for analyzing the probability of detecting gaming drug users. The analyses demonstrated that command monthly test rates and strategies can have a dramatic impact on the expected number of months until detection of gaming drug users.

It is recommended that this approach be used as part of the Navy's Drug Policy Analysis System (DPAS). Additionally, it is recommended that models be developed for estimating the cost to the Navy of an undetected drug user as a function of the length of time undetected. These costs, in conjunction with methodologies developed in this report, should be an essential input to the formulation of models to assist in the development of an optimal Navy drug testing program.

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